Coordination without Explicit Cooperation: 
Monetary-Fiscal Interactions in an Era of Demographic Change *

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Abstract: Early work on the theory of economic policy stressed the importance of accounting for the interactions between fiscal and monetary policy. Tinbergen, and Cooper, taught us that there would be costs in missed targets, instability, and protracted imbalances if this was not done. Yet most models we use today treat fiscal or monetary policies as if they operated alone. This paper reviews the advantages of recognising those interactions. We consider three possibilities: fiscal leadership (in the sense of a longer term pre-commitment), monetary leadership, and simultaneous decision making, each underpinned by independence at the central bank. Temporal separation is important because it creates an opportunity for punishment by the follower (a result from asynchronous games). Making fiscal policy lead therefore provides fiscal precommitment, and the best results for output, inflation and fiscal balances. In particular it ensures fiscal sustainability, without the need for arbitrary and easily evaded numerical rules. We show these results are proof against override by rational governments; and robust to market reforms that flatten the Phillips curve, or globalisation and the changes in savings caused by the ageing problem.

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1. Preface

This paper reviews the framework for setting fiscal and monetary policies in an expanded Euro area. It uses a model that predicts that the present policy arrangements will inevitably face difficulties in enforcing limits on the use of fiscal policy, whether in the form of the Stability and Growth Pact or some replacement mechanism. But equally they will be unable to ensure sustainable fiscal policies, and hence sustainable public finances, if they do not enforce those limits. The way out of this dilemma is to create a fiscal regime based on debt targets, withdraw target independence from the central bank, and replace it with fiscal commitment to compensate.

The key to these results is the implicit coordination that arises between the fiscal policies directed at long term debt targets and active monetary policies aimed at short term stabilisation; and then to use that coordination to manage the interactions that emerge between fiscal and monetary policies. Given that framework we show how the different policy-making institutions can retain different priorities, and hence individual policies that are internally consistent, while also maintaining a degree of flexibility that allows them to deal with problems as they arise. At the same time policy makers need to be independent of external influences, and short term political pressures in particular, so that their policies remain consistent in the pursuit of the goals that they, or society, have set for them.

In the European context, this means examining vertical coordination between the Central Bank, and the national fiscal authorities taken as a group. We therefore abstract from the horizontal coordination between the fiscal authorities (although section 4.1 and footnote 11 show how they can be fitted into this analysis) because the focus is on comparing different vertical coordination schemes in terms of their outcomes, policy mix and ability to ensure fiscal sustainability. We reach four general conclusions:

1) That fiscal leadership, of the kind described above, is the only regime that ensures long term fiscal sustainability.

2) Fiscal leadership is superior to other coordination regimes with an independent central bank on other performance criteria. Leadership here means that governments, having cre-

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1 Hughes Hallett and Scott (2004) show that the distinction between vertical and horizontal coordination is a natural part of any federal or devolved system of governance.
ated an independent central bank, adopt mechanisms that pre-commit them to a particular line of conduct for fiscal policy (say a Stability Pact with debt targets) and let the Central Bank deal with short run stabilisation. It is in their interest to adopt that regime.

3) This finding is robust to changing behavioural relations stemming from globalisation and structural reform, or to uncertainties in those relationships, and to increasing demand or supply shocks.

4) More unexpectedly, the advantages of fiscal leadership are jeopardised if the Central Bank is given target independence in addition to instrument independence since the fiscal authorities will then pursue their own inflation-output trade-offs by trying to offset the additional monetary rigour with a looser fiscal policy – which means enforcing a Stability Pact will become steadily more difficult and an economic populist government is bound to emerge at some point. But the other regimes also face the same difficulty. So target independence is not helpful in any regime because it makes the enforcement of fiscal restrictions more difficult and hence prejudices the sustainability of public finances.

2. The case for a change in the current framework

Received wisdom for the existing European policy framework says that the Central Bank should be independent of outside forces (and of exposure to political pressures in particular), and that national fiscal policies should be left free to account for and address the particular circumstances within each member economy. The difficulty with this idea is that an independent monetary authority necessarily implies an independent fiscal authority – which, in the European context, means a fiscal authority subject to political and electoral pressures.\(^2\) That implies increasing demands for fiscal policy to meet the preferences of local populations since monetary policies (being common to all) cannot. Fiscal policy will also be increasingly set to reflect that population’s legitimate interest in seeing that public money is spent on the goals, and to the extent, that it wants. Again the electoral mechanism will be the force behind this. As a result, fiscal and monetary policies are likely to, if not conflict, be at least used to blunt the unwanted impact of the other on their

\(^{2}\text{This has the implication that rational voters will drive governments harder to achieve what they have been elected to achieve, even if that conflicts with the policies pursued by the central bank. See Demertzis et al (2004) for a formal demonstration of this proposition, and for an analysis of the economic consequences.}
own preferred targets. That will lead to poor and possibly unstable outcomes. Indeed, that appears to be the story of the Stability and Growth Pact (SGP): it was first put in to rule out such uncoordinated and undisciplined behaviour. But under popular pressure, it was violated widely, as this argument suggests it would, and few counties now show any lasting commitment to adhering to it, even in its revised version.

Something needs to change in the current framework therefore. We need to reduce the self-interest in the one-to-one (policy to target) assignments of the current policy framework, given that there is no one-to-one separation of their effects on the policy targets. That suggests we need to find a way to create greater coordination (or at least consistency) between the fiscal and monetary policy makers, without introducing the need for face-to-face negotiations over the precise measures to be taken in any particular case that could compromise the independence of the policy makers to take whatever actions are thought necessary in the circumstances.

To do this we need to provide a less than exclusive *intertemporal* assignment of policy goals to policy makers – instead of the one-to-one “within period” assignments as currently imposed. The reason is that within period assignments mean that policies must compete to satisfy their own priorities within that time period, increasing the danger (or degree) of conflict between them. Each will end up trying to offset the effects of the decisions of the other, in a vain attempt to reach their own goals since they know they have no other opportunity to do so before the next round of conflict with their rival.

But if there is temporal separation, such as would happen if fiscal policies prioritise long term expenditure goals and monetary policy concentrates on short term stabilisation and inflation control, then there will be less conflict and each policy maker can take into account the existing or predicted stance of the other. There is also the threat that an undisciplined (or inappropriately selfish) move by one player will be punished, and will be anticipated to be punished, according to the preferences of the other by subsequent actions from the other player. Yet each player gets their chance to implement their own preferred policies without direct opposition of the other. This implied process of action and threatened counteraction, in effect a “negotiation over time”, leads to an equilibrium in which
policies are better coordinated than when the resolution has to be achieved by confrontation. A convenient way to set this implicit negotiation process in motion is to impose a debt target on fiscal policy making. Being a stock not a flow, a debt target implies a significant carry over from period to period and therefore forces a temporal difference (and a longer term view) onto fiscal planning not evident in monetary policy making. Also, being a moving total, debt is a better indicator of the structural position of the economy than deficits which vary, and can be manipulated, over the economic and political cycles. But there is no compromise on the independence of the central bank to make monetary policy as it sees fit. Similarly, there are no physical restrictions preventing fiscal policy makers from intervening with the policy priorities chosen by the public that elected them.

Fiscal-monetary interactions are just one example of where the interactions between the impacts of policies can change the outcomes from what one might expect from an analysis of the individual policies one by one. We touch on other examples here: the interaction between fiscal policy and the ability to carry out structural reform; or between labour market policies and social security, or between labour market reforms and supply side policies. This is an old theme, but we are giving it new European clothes here. Both Tinbergen (1956) and Cooper (1969), in their contributions to the theory of economic policy, stressed that difficulties will arise when policies are not coordinated properly; and that the consequence will be increased instability, and a inferior performance as each policy maker struggles to overcome the spillovers from decisions made by others while trying to get the best for his own targets within the given period. The result: larger deviations from target, more time spent away from target, and a slower return to target. Our framework allows better coordination without any loss of independence for the policy makers.

3. When is co-ordination among policies helpful?

Co-ordination involves some form of negotiation or joint decision making in which policy makers choose their policies away from the policies they would have chosen if left to themselves and their own private interests. Coordination therefore takes into account the

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3 Since structural reforms also have cross-country repercussions, the question arises as to whether a lack of co-ordination among supply-side policies (in R&D, skills and training, competition policy, etc.) is wise.
possibility that helping others achieve a stronger performance may put each policy maker
in a stronger position to benefit from better outcomes himself. Likewise, coordination
eliminates the costs imposed on others (in the sense of making progress towards their tar-
ggets more difficult) by purely selfish behaviour. If each player can be made better off by
eliminating the costs imposed on others, then the economy must be made better off.

Exactly the same arguments apply to independent, self-interested decisions made by the
policy making institutions (monetary or fiscal) within one country as they do between
countries, perhaps with added force since their decisions will affect all markets rather
than just those markets which deal in traded goods and foreign investment. Coordination
here would reduce the spillovers which the single minded pursuit of the goals of one in-
stitution would impose on the targets and policies of another. A tight monetary policy to
maintain low inflation might cause larger output fluctuations or slower growth, for exam-
ple; or expansionary fiscal policies designed to improve employment levels might cause
inflation. Closer inter-institutional coordination would lower the incentive to create such
spillovers, and hence lower the need for counteractive policies to absorb those spillovers.

Explicit coordination therefore involves a negotiated bargain in which the outside option
- the point to which the participants can threaten to return if negotiations break down - is
the narrowly self-interested, uncoordinated solution noted above. Thus bargaining power,
the inverse of how much one would lose if the bargain broke down, will likely determine
how far each policy maker’s private interests are met in the final coordinated solution. To
take place (to command assent), the coordinated policies must be incentive compatible:
i.e. offer some gains to each participant individually, rather than just on average, over the
best that each could hope to achieve in the absence of any coordination.

Coordination will be at its most effective when the policy spillovers between economies
and onto non-assigned targets are strongest; or where one policy maker has comparative
advantage (effectiveness) in reaching certain targets with one instrument, and another has
comparative advantage with another. In the latter case, coordination brings gains because
it allows the reallocation of policy effort to those with comparative advantage, even if
that means making less effort at home with the (relatively) less effective instrument. Co-
ordination is therefore most valuable when economies are structurally different (including different dynamics); and when the policy arenas in which the instruments are operating involve different transmission mechanisms, so that comparative advantage or economies of scale in policy effectiveness can be exploited more fully. Inter-institutional coordination is an example *par excellence* of this latter case.

Harmonization is different. Indeed, harmonisation may not even be helpful because it does not allow comparative advantage to be exploited correctly when countries have different structures or policy responses. It is entirely possible that harmonisation could lead governments to enact policies sufficiently far from what would be efficient for their situation, such that the outcomes become incentive incompatible compared to what those governments would have chosen for themselves. If that were the case, the government in question would inevitably want to withdraw from the joint decision making regime unless its economy were identical to every other (participating) economy.

These points emphasize that coordination will become important where there is a mutual dependence between the targets of economic policy, and where there are asymmetries in structure or transmission that govern how those targets respond to changes. Most often we think of these asymmetries as differences in the way that targets respond in different countries; implying that cooperation between countries will be the key issue. But asymmetries also imply that coordination between the different institutions of policy making within a country or currency zone will matter. In fact, it is likely that inter-institutional coordination will be the more important element. Hence, in Europe, to arrange coordination between policies operating in different policy arenas, for example between monetary and fiscal policies, or labour markets and structural reform, may prove more effective than greater coordination between countries in any one arena.

4. Co-ordination in EMU: which areas are relevant?

4.1 Monetary policy

If there are problems with monetary policy, they come from instability in the external value of the Euro; or from the asymmetric effects of a common monetary policy because
different countries have different transition mechanisms or lack a common business cycle. The first problem would call for coordination with US policies and those in Asia. This lies beyond the remit of this paper. The second is a matter of how well the asymmetric effects can be compensated locally by more flexible fiscal policies, or by more flexible market adjustments. They are better dealt with under the headings of fiscal policy and structural reform.

There is one further issue however: a common monetary policy is, strictly speaking, suboptimal because it aims to reduce the deviation of the average EU inflation rate from target, not the average of the individual deviations. It therefore omits any concern for the variability or distribution of those deviations across countries. In an inflationary period, those with below average inflation are penalised and forced to tighten as much as those with above average inflation. Similarly, in a recession, those with above average inflation must loosen just as much as those below average. The question is whether policy would be better served if the differences in national circumstances were also to enter the policy calculations. One way would be to ask policy makers react to the variance and skews in inflation rates. A second would be to increase the weight of countries in the pan-EU inflation index in proportion to their deviation from the ECB’s target (Benigno 2004). But the third and most obvious would be to ask fiscal policy makers to adjust their fiscal stance to compensate for national differences, expanding where inflation is below average and contracting when it is above. But that again requires coordination between fiscal and monetary policies.

This need for extra flexibility at a national level could be provided by greater wage and price flexibility of course, in which case fiscal flexibility is less important. That shows structural reform may have an important impact on fiscal policy. But Europe’s markets are notoriously short on flexibility, especially in the labour market. Consequently, there is a need for inter-policy coordination in those two arenas, similar to that between fiscal and monetary policies. If that is not feasible, a common fiscal policy at the union level with the flexibility to reduce the European cycle (so national stabilisation is not so pressing), and regional deviations that sum to zero to deal with country specific asymmetries, is likely to be the best substitute.
4.2 Fiscal policy and the Stability Pact

The Stability and Growth Pact, introduced in 1997 as a means of restraining the impact of excessive deficits and public sector debt on economic performance, has not been entirely successful. It remains operative on paper – complete with its excessive deficit procedure, sanctions for violators and surveillance mechanisms. But it remains ineffective in practice, for at least three reasons.

The first is that it has proved to be unenforceable. The reason is that the Amsterdam treaty defines a country to have an excessive deficit if that country’s fiscal deficit exceeds 3% of its gross domestic output, and if the Council of Ministers judges it to be so. This leaves open the possibility that the Council of Ministers will judge the deficit not to be excessive on the grounds that the excess is either excusable, or mismeasured, or has appeared for reasons beyond the accused government’s control.\(^4\)

A second reason why the Pact will remain inoperative is political. A necessary condition for enforcement – that the Council of Ministers declare a country to be in violation of the excessive deficit procedure – inevitably involves “sinners sitting in judgement of sinners” since the Council will include representatives of the violating government and unanimity is required in matters of taxation and fiscal policy. Even if unanimity is taken away, there is little incentive for countries to vote to support an excessive deficit decision and sanctions. The accused have a natural incentive to work for a veto. Others have a natural incentive to use that veto, whether they have excessive deficits or not, on the argument that “there, but for the grace of God, go I” or that “it will be our turn next, so a veto now will bring a veto in our favour next time”. It therefore seems unlikely that the Stability Pact, as currently construed, would ever produce a judgement that a country was in violation of the deficit rule and should be sanctioned – and especially if it involves a large country, too large to fail in the context of the Euro economy.

The third reason has to do with incentives. If there was little incentive for governments to keep to the Stability Pact in its original form, then there is no obvious reason for them to

\(\^{\text*}4\)All too easily argued given the evident difficulty in estimating whether an underlying structural deficit is excessive or not, a necessary test to make this judgment (Hughes Hallett, Lewis and Kattai, 2007).
keep to the Pact after it was eased in 2005. Obviously they would find it easier to do so: but that is not the same as saying that they would want or try to do so. A violation seen as permissible under the old Pact can be condoned even more easily under the new version⁵.

Given this lack of an effective fiscal restraint mechanism, in EMU at least, it would pay all parties (governments, central bank) to agree to adopt some coordinating procedures when making their policy choices so that each may have the ability to restrain the other directly; and so that each has the means to threaten retaliation if there is an abuse.

4.3 Debt Targeting

Other solutions to the Stability Pact problem have also been considered – setting debt targets (a debt rule), a golden rule (balancing the budget for non-investment expenditures only), targeting cyclically adjusted budget deficits, and substituting “soft” for “hard” targets. It turns out these options are too weak numerically to make a material difference (Fatas et al. 2003). Debt targets are an exception. They have the advantage of being a stock and not a flow. That implies a degree of persistence -- especially in countries with high levels of public debt. Debt targets can therefore be used to pre-commit fiscal policies into the future, more so in countries where fiscal policies have been lax in the past, to a path consistent with the expected stance of monetary policy and the goals of sustainability in public finances and limited spillovers on others.⁶

In this paper I make much of the fact that fiscal policy lends itself naturally to long term objectives, and that this provides an element of leadership which can be combined with independent monetary policies directed at short run (demand management) objectives. That creates a basis for rule-based coordination between policy makers without the need for explicit negotiation. Each policy would be operating according to comparative advan-

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⁵ The following can now be excluded from the excessive deficit calculations: public investment; defence; development aid; conversion of pension funds to a fully funded basis; the costs of structural or market reforms; expenditures on European integration; also all deficits when growth is negative or below potential, or where deficits are being reduced by at least 0.5% of GDP a year.

⁶ To make fiscal policy follow a debt target also has a strong theoretical justification. Persson et al (2006) show that the optimal balance of fiscal and monetary policies will be achieved if the economy maintains a debt stock and maturity structure that exactly balances the marginal benefits of a surprise inflation with its marginal costs. The only question is whether superior results can also be obtained if only the debt stock is targeted, rather than the full maturity profile. The results in Table 1 below demonstrate that they can.
tage: fiscal policies directed at the goals of public finance, and an independent monetary policy to underwrite the commitment to credible stabilisation policies. The leadership element comes from the fact that fiscal policies typically have longer run targets (the sustainability of public finances, low debt), and are not easily reversible (public services, social equality), and are not easily used for stabilisation if consistency across time and different types of policy is to be maintained. Moreover any policy, independent or not, must be conditioned on the state of the economy and other policies. Improved coordination then follows because policies can be adjusted to reduce the expected constraints (externality costs) imposed by one set of policies on another. If the costs imposed on one policy are eased, then that policy can be relaxed which allows the first policy to be scaled back and not cause the externality in the first place. Less conflict and more favourable operating conditions lead to more efficient policies and better outcomes.

However there are automatic stabilizer effects in any fiscal policy framework, implying that monetary policy must condition itself on the expected fiscal stance at each point. This puts monetary policy in a follower’s role, unavoidably. This is helpful: it allows the economy to retain the benefits of an independent monetary policy, but also ensures a certain degree of flexibility and coordination between the two sets of policy makers. The result is a Pareto improvement over the non-cooperative solution, without any reduction in the central bank’s ability to act independently on its shorter run objectives.

Second, we suppose soft targets would probably work better for the following reasons. Policy making is less likely to be disabled by arguments over the precise definition and measurement of the target, or the arbitrary nature of a numerical limit. In addition, soft targets introduce flexibility into policy making – so that the procyclicality of hard targets is reduced, along with the tendency of rigid targets to block reforms whenever the latter have short run costs. Soft targets can also accommodate the positive effects of a deficit, and the desire to allow differences in national priorities when simplicity and fairness suggest uniform limits should be imposed in the long run.

Third, we can expect debt targets to work better than deficit limits because they focus on avoiding the ultimate risk, unsustainable public finances. Moreover, a soft target version
allows policy makers to trade off good years against bad (in effect, because the target is a stock not a flow, this produces a cyclically adjusted deficit rule without the difficulties of having to calculate the cyclically adjusted deficits accurately). Fourth, and again because the target is a stock and therefore persistent, a debt rule gives policy makers a greater incentive to obey the rules: i) to preserve freedom of action in the future; ii) to save for a rainy day at the top of the cycle and remain “within target” for the downturn; and iii) because the persistence in such a target makes it possible to create a reputation and credibility. A debt target therefore creates fiscal leadership by giving the policy makers a commitment technology with a long term aim and a slowly moving stock variable as target. The fiscal authorities will be obliged to set their longer term plans first; and we get the precommitment we need – but only if monetary policy follows (with a shorter horizon) to provide the threat to opportunistic fiscal policy making thereafter.7

Finally, to reflect the fact that we live in a world where shocks are becoming more global (as opposed to local or asymmetric) comparing different regimes with vertical coordination needs to take precedence over those with horizontal coordination. Also we should adopt a model that allows a shift in the mix of shocks: from demand to supply shocks to reflect increasing oil and food prices, outsourcing to emerging economies, or changes in demographics as populations age. To a large extent, these points motivate the choice of model and regimes studied in this paper: vertical coordination, pervasive supply shocks, and multiple interpretations for the model parameters (see section 8).

5. A Model of Fiscal Pre-Commitment

5.1 The Model

The key question is: would governments ever agree to pre-commit their fiscal policies? Do they have an incentive to do so; would economic performance improve if they did?

7 The results in section 6 and Table 1 show that this statement is true, but a formal proof is given in Libich et al (2007). The proof uses the theory of asynchronous games, where each player can intervene only once in so many periods. If the length of commitment interval is different for each player, then the one with the shorter horizon can always intervene to punish the other unopposed in the last few periods before the lowest common multiple of their commitment periods (where the intervention cycle repeats and both are back at their threat point). This threat of punishment, without an opportunity to retaliate before the game is repeated again from a common starting point, means that the player with the longer commitment interval will become precommitted.
Can independent policy making lead to sustainable fiscal policies? Or do some regimes need fiscal constraints to guarantee the sustainability of public finances?

To answer those questions, we extend the model used in Hughes Hallett and Weymark (2005, 2007) to create a steady state dynamic general equilibrium model in which monetary policy interacts with, but could be undermined by, fiscal policy. For exposition purposes, we suppress the spillovers between countries and focus on the following three equations to represent the economic structure of any one country:  

\[ \pi_t = \pi_t^e + \alpha y_t + u_t \]  
\[ y_t = \beta (m_t - \pi_t) + \gamma g_t + \epsilon_t \]  
\[ g_t = m_t + s (b y_t - \tau_t) \]

where \( \pi_t \) is inflation in period \( t \), \( y_t \) is the growth in output (relative to trend) in period \( t \), and \( \pi_t^e \) represents the rate of inflation that rational agents expect to prevail in period \( t \) conditional on the information available at the time expectations are formed. Next, \( m_t, g_t, \) and \( \tau_t \) represent the growth in the money supply, government expenditures, and tax revenues in period \( t \); and \( u_t \) and \( \epsilon_t \) are random disturbances which are distributed independently with zero means and constant variances. All variables are defined to be deviations from their steady state (or equilibrium) growth path values, and we then treat trend budget variables as pre-committed and balanced. Deviations from the trend budget are therefore the only discretionary fiscal policy choices available. The coefficients \( \alpha, \beta, \gamma, s, \) and \( b \) are all positive by assumption. The assumption that \( \gamma \) is positive is sometimes controversial.\(^9\) However, the short-run impact multipliers derived from Taylor’s (1993) multi-country model provide empirical support for this assumption (as does HMT 2003).

\( \text{\footnotesize To derive (1)-(3) from a multicountry model, I have to impose a blockwise orthogonalisation to generate independent semi-reduced forms for each country. The disturbance terms may then contain foreign variables, but they will have zero means so long as those countries remain on their long run (equilibrium) growth paths on average – all variables being defined as deviations from their equilibrium growth paths.} \)

\( \text{\footnotesize Barro (1981) argues that government purchases contract output. Our model, by contrast, treats fiscal policy as important because: (i) fiscal policy is widely used to achieve public service objectives; (ii) governments cannot pre-commit monetary policy with any credibility if fiscal policy is not pre-committed (Dixit and Lambertini, 2003a); and (iii) Central Banks, and the ECB in particular, worry intensely about the impact of fiscal policy on inflation and financial stability (Dixit 2001).} \)
According to (1), inflation is increasing in the rate of inflation predicted by private agents and in output growth. Equation (2) indicates that both monetary and fiscal policies have an impact on the output gap. The micro-foundations of the aggregate supply equation (1), originally derived by Lucas (1972), are well-known; and McCallum (1989) shows how aggregate demand equations like (2) can be derived from a standard, multi-period utility-maximisation problem. Both are derived in detail by Dixit and Lambertini (2003a).

Equation (3) meanwhile is a budget constraint to impose the restriction that fiscal policy must be sustainable both politically (provide enough redistribution, public services) and in an accounting sense (public debt must remain sustainable, a transversality condition).

5.2 Two interpretations of the budget constraint:

(i) Social equity or income redistribution.

Equation (3) describes the government’s budget constraint (Hughes Hallett and Weymark 2005 give an explicit derivation). Different attitudes to redistribution are the feature that normally differentiates political parties and their fiscal policies. Monetary authorities, by contrast, are seldom granted the mandate or the tools to address issues of social equity or income distribution. We therefore allow discretionary tax revenues to be used for redistributive purposes, but retain discretionary expenditures for enhancing output\(^{10}\). Both are deviations above (or below) their balanced budget equilibrium paths. But the additional uncovered expenditures can only be financed by creating money, \(m_t\), or borrowing post-tax earnings from the public: \(s(b y_t - \tau_t)\). Since the former is the exclusive preserve of the central bank, fiscal policy will focus on the latter as a contribution to discretionary spending limited by the government’s ability to finance that spending.

We further assume that there are two types of agents, rich and poor, and that only the rich have sufficient savings to buy government bonds. On this view, \(b\) would be the proportion of pre-tax income going to the rich and \(s\) the proportion of after-tax income that the rich allocate to saving. Only the rich save, and only the post-tax earnings of the rich can be saved and lent; hence the discretionary expenditures available to the government must be constrained by the discretionary tax revenues the government decides to devote to re-

\(^{10}\) See Hughes Hallett and Weymark (2004a, 2005, 2007).
distribution and its public spending objectives. That explains the signs of $\gamma$ and $\tau$, in (3)\textsuperscript{11}. The tax revenues can then be used by the government to redistribute income from rich to poor, either directly or through the provision of public services. This model therefore has output enhancing expenditures $g$, and discretionary fiscal transfers $\tau$. Both are financed by aggregate tax revenues; that is, from both discretionary and trend revenues. Expenditures above those revenues must be financed by the sale of bonds.

(ii) \textit{An explicit debt target.}

Now we take a different view. Equation (3) remains the government’s budget constraint. But we define $b$ to be the current public debt to GDP ratio; $\tau$ to be the revenues that the government proposes to devote to paying down the existing stock of debt in period $t$; and $s$ to be the proportion of any remaining increase in the debt stock (after transfers to pay it down) that the government expects to pay out in current expenditures this period. It may not be obvious that a government could spend out of an existing, and hence previously committed, stock of debt. However we are targeting a debt \textit{ratio} here, not debt itself. If $b = B/Y$, where $B$ is the stock of nominal debt and $Y$ is the level of national income, then $\Delta b = [\Delta B - b\Delta Y]/Y$ where $\Delta x$ denotes a change in $x$ per unit of time. But $b\Delta Y = by_t$, since, by definition, $y_t$ is the deviation of $Y_t$ from its own steady state path. Hence $by_t$ represents the amount of additional debt that could be created and spent without the debt to GDP ratio rising when national income rises or falls.

We now suppose governments spend only a proportion $s$ of that quantity, after taking into account the revenues they plan to use to pay the debt stock down. This is allows us to introduce a debt target into the government’s objectives, such that $\tau$ is chosen to reduce the debt ratio from $b$ to some target level, $\theta$ say.\textsuperscript{12} The implication is that $s(by_t - \tau)$ represents only that part of the debt increase which the government plans to spend for short term stabilisation purposes. This follows from equations (2) and (3). The rest will be used for public spending, investment, structural reform, debt relief or other long term

\textsuperscript{11} Since (3) defines the spending that can be financed, $g$, will be procyclical. Fiscal policy therefore has long run targets, but not a stabilization role.

\textsuperscript{12} This shows that we are solving a dynamic steady state problem, not a period by period case.
commitments. We might therefore regard \( s \) as the proportion of new fiscal expenditures going to automatic stabilisers (in the absence of discretionary stabilisation), and \((1-s)\) the proportion going to longer term goals. Hence we might expect \( s \) to be about 0.5 for the Euro economy (EC, 2002); or 0.33 for an economy like the UK (HMT 2003).

5.3 Government and Central Bank Objectives

We allow the government and central bank to differ in their objectives and priorities. We assume that the government cares about inflation, output growth, and the provision of public services (and hence the size and sustainability of public debt); whereas the central bank, if left to itself, would be concerned only with the first two objectives; and possibly only the first one. We also assume that the government has been elected by majority vote, so that the government’s loss function reflects society’s preferences to a large extent.

Formally, the government’s loss function is given by

\[
L_t^g = \frac{1}{2} (\pi_t - \hat{\pi}_t)^2 - \lambda_1^g \Delta_y_t - \frac{\lambda_2^g}{2} [(b - \theta) y_t - \tau_t]^2
\]

(4)

where \( \hat{\pi}_t \) is the government’s inflation target, \( \lambda_1^g \) is the relative weight or importance that the government assigns to output growth,\(^{14}\) and \( \lambda_2^g \) is the relative weight or degree of pre-commitment assigned to the fiscal sustainability or social equity rule. The parameter \( \theta \) represents the target value for the debt or deficit to GDP ratio which the government would like to reach: hence \((b - \theta) y_t\) becomes the target for its discretionary revenues \( \tau_t \). All other variables are as previously defined. We take larger values of \( \lambda_2^g \), \( \lambda_2^g > \max[1, \lambda_1^g] \), as defining a “hard” debt rule; and small values, say \( \lambda_2^g < \min[1, \lambda_1^g] \), as a “soft” debt rule.

\(^{13}\) Multiple fiscal policy makers can be accommodated in this framework if they have identical target values for inflation, output and debt (Dixit and Lambertini, 2003b). That gives us the option to apply our results to a currency union (Europe), a federal union (US), or a regime of devolved regional governments (the UK).

\(^{14}\) Barro and Gordon (1983) also adopt a linear output target. In the delegation literature, the output term in the government’s loss function is usually represented as quadratic to reflect an output stability objective. In this model, the quadratic term in debt/deficits allows monetary and fiscal policy to play that stabilization role. A better explanation of the linear term in (7) would be that it reflects the presence of monopolistic competition in the markets, which lowers output and which the fiscal policy makers attempt to counteract. It also captures the policy makers’ revealed preference for not saving for a rainy day.
The objectives of the central bank, however, may differ from those of the government. We model them as follows:

$$L_i^{cb} = \frac{1}{2} (\pi - \hat{\pi})^2 - (1 - \delta)\lambda^{cb} y_t - \delta \lambda^{yg} y_t + \frac{\delta \lambda^{gb}}{2} [(b - \theta)y_t - \tau_t]^2$$

(5)

where $0 \leq \delta \leq 1$, and $\lambda^{cb}$ is the weight which the central bank assigns to output growth. The parameter $\delta$ measures the degree to which the central bank is forced to take the government’s objectives into account. The closer $\delta$ is to 0, the greater is the independence of the central bank in making its choices. And the lower is $\lambda^{cb}$, the greater is the degree of its conservatism in making those choices.

In (4) we have defined the government’s inflation target as $\hat{\pi}$. To specify the same inflation target in (5) as in (4) is to imply that the bank has instrument independence, but not target independence. It is easy to relax that assumption and allow the central bank to choose its own target, as the ECB does. But, as I show in Hughes Hallett (2005), there is no advantage in doing so since the government will simply adjust its priorities (to give the bank less independence) to counter the unwanted lower inflation target. Hence, only if the bank is free to choose the value of $\lambda^{cb}$ as well as $\hat{\pi}$, do we get the extra advantage. That will be the definition of target independence in this paper.

6. Policy Regimes

We characterize the strategic interaction between the government and the central bank as a two-stage non-cooperative policy game in which the structure of the model and the objective functions are common knowledge. In each regime, the first stage involves setting the constitutional parameters $\delta$ and $\lambda^{cb}$, and the second involves choosing policy values for $m_t, g_t$ and $\tau_t$. Unless the central bank is granted target independence, it must accept an inflation target $\hat{\pi}$ set by the government.

We consider three different institutional regimes. The first is a benchmark in which there are no restrictions on the competition between fiscal and monetary policy makers at the second stage, but the constitutional parameters are jointly chosen by the government (or society) at the first. This we term “simultaneous policy moves”. The second regime is
“fiscal leadership”. Here the government chooses both $\delta$ and $\lambda^{cb}$ at the first stage, but the fiscal authorities choose $g_i$ and $\tau_i$ ahead of $m_i$ in a Stackelberg game at the second. The third regime is “monetary leadership”, where the government chooses $\delta$ and $\lambda^{cb}$ at the first stage, but the monetary authority sets $m_i$ ahead of $g_i$ and $\tau_i$ at the second. In each case, pre-commitment has its usual meaning: policy makers face no incentive (would not wish) to change their policies to something more advantageous in the absence of unforeseen shocks.

Each regime can then take two forms: instrument independence and target independence. Under instrument independence, the government chooses $\delta$ and $\lambda^{cb}$ as above; and the policy makers choose $m_i, g_i$ and $\tau_i$ according to whichever game they are in at the second stage. But under target independence, the government chooses $\delta$ and the central bank $\lambda^{cb}$ and $\hat{\pi}^{cb}$ (its inflation target) at the first stage, while policy makers choose $m_i, g_i$ and $\tau_i$ according to the relevant game at the second stage. Fiscal leadership can also take a (third) hybrid form in which the bank chooses $\lambda^{cb}$ at the first stage; but the government sets $\hat{\pi}^{cb} = \hat{\pi}$ to ensure a measure of coordination. This is called the Bank of England variant, although several other prominent central banks follow the same model.

6.1 The Simultaneous Policy Moves Regime

In this regime, the government first chooses the parameters $\delta$ and $\lambda^{cb}$. Then the second stage is a Nash equilibrium in which the government and monetary authority set their policy instruments, $g_i$ and $\tau_i$, for the government and $m_i$ for the central bank, given the values of $\delta$ and $\lambda^{cb}$ just chosen, the inflation target $\hat{\pi}$, and after the shocks $u_i$ and $\epsilon_i$ appear. Private agents understand the game and form rational expectations about future prices and policies before the second stage: that is, before the policy makers implement their policies, and before the shocks are revealed, but after the parameters $\delta$ and $\lambda^{cb}$ have been set.

We solve this game by solving backwards: for the policy choices first; and then substituting the results back into (4) to determine the optimal institutional parameters $\delta$ and $\lambda^{cb}$.
The details of this solution, and the related solutions with fiscal and monetary leadership, are given in the appendix. From the non-cooperative equilibrium at stage 2, we get:

\[ \pi_i(\delta, \lambda^{cb}) = \hat{\pi} + \frac{(1 - \delta)\beta \phi \lambda^{cb} + \delta(\beta \phi + \gamma \Lambda)\lambda^g_1}{\alpha[\beta \phi + \delta \gamma \Lambda]} \]  
\[ \gamma_i(\delta, \lambda^{cb}) = -u_i/\alpha, \]  
and \[ \tau_i(\delta, \lambda^{cb}) = \frac{(1 - \delta)\beta \psi(\lambda^{cb} - \lambda^g_1)}{[\beta \phi + \delta \gamma \Lambda]\lambda^g_2} - \frac{(b - \theta)u_i}{\alpha} \]  

where \( \phi = 1 + \alpha \beta - \gamma \theta \) and \( \Lambda = 1 + \alpha \beta + \beta \theta \).  

Evidently \( \Lambda \) is positive; and we can assume \( \phi \) to be positive as well, for the reasons given in the appendix. In fact, numerical evaluations for ten of the more advanced OECD economies place \( \phi \) close to unity which confirms that assumption. Notice also that the demand shocks, \( \epsilon_i \), play no role in this solution.

Substituting (6)-(8) back into (4), we get an optimized value of the government’s objective function in terms of \( \delta \) and \( \lambda^{cb} \) from which the stage 1 decision can be obtained. Minimizing again with respect to \( \delta \) and \( \lambda^{cb} \) yields two pairs of first order conditions which are satisfied by two different sets of values for \( \delta \) and \( \lambda^{cb} \). Both are satisfied when \( \delta = 1 \) and \( \lambda^{cb} = \lambda^g_1 \). That solution describes an entirely dependent central bank. However, it is inferior to the second solution where \( 0 \leq \delta < 1 \) and \( \lambda^{cb} \neq \lambda^g_1 \). In this case the optimal value of \( \delta \) is given by (A14) of the appendix:

\[ \delta = \frac{\beta \phi^2 \lambda^{cb} \lambda^g_2 + \alpha^2 \gamma^2 s^2 \beta (\lambda^{cb} - \lambda^g_1)}{\beta \phi^2 \lambda^{cb} \lambda^g_2 + \alpha^2 \gamma^2 s^2 \beta (\lambda^{cb} - \lambda^g_1) - \phi(\beta \phi + \gamma \Lambda)\lambda^g_2 \lambda^g_2} \]  

Out of these two solutions, we can check which yields the lower welfare loss, as measured by the government’s (society’s) loss function, using (4). Substituting \( \delta = 1, \lambda^{cb} = \lambda^g_1 \), and (6)-(8), into (4) results in a greater expected loss (lower welfare) for the government than substituting (10) together with (6)-(8): see equations (A15) and (A16). That confirms that the second solution is the optimal choice. Independence and conservatism is always better if the government has social or fiscal sustainability objectives.

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15 See Hughes Hallett and Weymark (2007)
The target independence version of this regime, where the central bank chooses its own inflation target and priorities, \( \hat{\pi}^{cb} \) and \( \lambda^{cb} \), is examined in Hughes Hallett and Weymark (2004b). It produces a different distribution of outcomes, but an inferior welfare result because the government retains the right to manipulate the value of \( \delta \). The only practical difference is that an extra term has to be added into each of (6) and (8), while (7) remains unchanged. It is easy then to check that this solution produces lower inflation, but reduces tax revenues whenever \( \hat{\pi}^{cb} < \hat{\pi} \). Since we must expect \( \hat{\pi}^{cb} \leq \hat{\pi} \), this means target independence in this regime is trading less fiscal sustainability for lower inflation. That produces an inferior welfare outcome because the central bank will always choose a value of \( \hat{\pi}^{cb} \) lower than \( \hat{\pi} \), but sufficiently close to it, to induce the government to choose \( \delta = 0 \). That minimizes the bank’s loss function, but leaves the government/society’s losses somewhere between those for the optimal independent central bank and the fully dependent one.\(^{16}\)

6.2 Fiscal Leadership

In this regime, the government still chooses the institutional parameters \( \delta \) and \( \lambda^{cb} \). But the second stage is a Stackelberg game in which the fiscal authority chooses \( g \) and \( \tau \), ahead of the monetary policy decision \( m \). But they do so knowing what the monetary policy rule for choosing \( m \) will be. The fiscal authorities therefore internalize, in their decision making, the anticipated reaction function of the monetary authorities. This time, the government and the monetary authority set their policy instruments, given \( \delta \) and \( \lambda^{cb} \) values determined in the knowledge that fiscal leadership will follow. Private agents understand the game and form rational expectations for future prices and policies.

Again we solve this game backwards: for the policy choices first, then by substituting the results back into (4) to determine the optimal institutional parameters \( \delta \) and \( \lambda^{cb} \). We get

\[
\pi_i(\delta, \lambda^{cb}) = \hat{\pi} + \frac{(1-\delta)\beta(\phi - \eta \Lambda)\lambda^{cb} + \delta(\beta\phi + \gamma \Lambda)\lambda^{cb}}{\alpha[\beta(\phi - \eta \Lambda) + \delta\Lambda(\beta \eta + \gamma)]}
\]

\[\gamma_i(\delta, \lambda^{cb}) = -u_i/\alpha,\]

\(^{16}\) Hughes Hallett and Weymark (2004b), section 4.4, gives the exact calculations.
\[
\tau_c(\delta, \lambda^{cb}) = \frac{(1 - \delta)\beta s(\beta \eta + \gamma)(\lambda^{cb} - \lambda_1^c)}{[\beta(\phi - \eta \Lambda) + \delta \Lambda(\beta \eta + \gamma)]\lambda_2^c} - \frac{(b - \theta)u_c}{\alpha}
\]

(13)

where \(\eta\) is given by equation (A22), and \(\phi\) and \(\Lambda\) are as before.

Substituting (11)-(13) back into (4), we can now get the stage 1 solution from a partly optimised expression for the government’s expected objective function. This part of the problem has a pair of first order conditions, which are satisfied by two real-valued solutions (plus two complex solutions which can be ignored). Both first order conditions are satisfied when \(\delta = 1\) and \(\lambda^{cb} = \lambda_1^c\). That describes a fully dependent central bank. However, it is inferior to the second solution: \(\delta = \lambda^{cb} = 0\). In that solution, the central bank is fully independent and exclusively concerned with the economy’s inflation performance.

Out of these two possibilities, the solution which yields the lowest welfare loss, as measured by the government’s (society’s) loss function, can be identified by comparing \(EL^s\) in (4) to the expected loss that would be suffered under the alternative institutional arrangement. Substituting \(\delta = 1\) and \(\lambda^{cb} = \lambda_1^c\) into (4), with (11)-(13), results in

\[
EL^s = \frac{(\lambda_1^c)^2}{2\alpha^2}
\]

(14)

But substituting \(\delta = \lambda^{cb} = 0\), with (11)-(13), into the right-hand-side of (4) yields

\[
EL^s = 0.
\]

(15)

Consequently our results show that, when there is fiscal leadership, society’s welfare loss is minimized when the government appoints independent central bankers who are concerned only with the achievement of a mandated inflation target and completely disregard the impact their policies may have on output.

**The Bank of England variant.** Many central banks in fact operate under an extended form of instrument independence: most prominently the Bank of England, the Swedish Riksbank, and the Reserve Banks of Australia and New Zealand, who are free to choose their own priorities (\(\lambda^{cb}\)), but must accept an inflation target set by the government. They may also face fiscal leadership established by fiscal constitution, budgetary practice or an override clause (Hughes Hallett 2007).
We analyse this variant using (11)-(13) since the second stage of the game, as a function of the stage 1 parameters $\delta$ and $\dot{\lambda}^{cb}$, has not changed. Moreover, since the government retains the right to choose $\delta$, the first order conditions $\partial EL^g / \partial \delta = 0$ are the same as in the instrument independence case; while $\partial EL^{cb} / \partial \lambda^{cb} = 0$ can be obtained by substituting (11)-(13) into (5) to obtain an expression for $EL^{cb}$. From this pair of first order conditions, we obtain two solutions: $\delta = 1, \lambda^{cb} = \lambda^g_0$ is still one solution, and $\delta = \lambda^{cb} = 0$ is still the other. Hence the Bank of England variant yields the same outcomes as fiscal leadership with more limited instrument independence. In addition, $EL^g = EL^{cb} = 0$ holds as before.

The target independence version. We now allow the central bank to choose its own inflation target $\hat{\pi}^{cb}$ as well. This case is more complicated because reworking the stage 2 outcomes, (11)-(13), shows that the expression for inflation, (11), will the term in $\hat{\pi}$ replaced by a new term which is no longer independent of the choice of $\delta$. That means the expressions for $EL^g$ and $EL^{cb}$ will have an additional term in their respective inflation components. But it is straightforward, if tedious, to compute that $\partial EL^g = \frac{1}{2}(\hat{\pi}^{cb} - \hat{\pi})^2 > 0$ and $\partial EL^{cb} / \partial \delta < 0$ at $\delta = \lambda^{cb} = 0$; and that $EL^{cb}$ and $\partial EL^{cb} / \partial \delta$ are both zero at the same point. That means the government will certainly choose $\delta > 0$, even if $\lambda^{cb}$ remains at zero, to offset the fact that the central bank would otherwise achieve $\bar{\pi} = \hat{\pi}^{cb}$, see (A29), when the government is trying to achieve a different (somewhat higher) inflation outcome. That means the inflation outcomes will be higher, by (11), and the expected revenues lower than in the instrument independence or Bank of England variants. As a result fiscal policy and public sector debt will become unsustainable, and the outcomes will be less satisfactory for the central bank ($EL^{cb}$ rises from its first best optimum of zero). Full target independence is therefore counterproductive in this case, because the coordination between fiscal and monetary authorities is broken.

6.3 Monetary Leadership

If instead the central bank has leadership and acts a Stackelberg leader, it will always have the ability to select its own inflation target $\hat{\pi}^{cb}$. But it may be constrained by statute, by convention, or by an override clause, in its independence and choice of priorities. If
that were the case, the central bank would have a reduced form of instrument indepen-
dence mirroring the Bank of England variant above.

If the government retains responsibility for $\delta$ and $\lambda^{cb}$, the game works as follows. Solving
backwards, the Stackelberg outcomes at stage 2 (as functions of $\delta$ and $\lambda^{cb}$) are:

$$\pi_i(\delta, \lambda^{cb}) = \frac{(\beta + \mu \gamma) \phi \hat{\pi}^{cb} + \delta \gamma (\Lambda - \mu \phi) \hat{\pi}}{(\beta + \mu \gamma) \phi + \delta \gamma (\Lambda - \mu \phi)} + \frac{(1 - \delta)(\beta + \mu \gamma) \phi \lambda^{cb} + \delta (\beta \phi + \gamma \lambda) \lambda_{i}^{g}}{\alpha[(\beta + \mu \gamma) + \delta \gamma (\Lambda - \mu \phi)]} \tag{16}$$

$$y_i(\delta, \lambda^{cb}) = -u_i / \alpha \tag{17}$$

$$\tau_i(\delta, \lambda^{cb}) = \frac{\alpha \gamma \phi \beta \mu \gamma (\hat{\pi}^{cb} - \hat{\pi}) + (1 - \delta) \gamma (\beta + \mu \gamma) s (\lambda^{cb} - \lambda_{i}^{g})}{[(\beta + \mu \gamma) \phi + \delta \gamma (\Lambda - \mu \phi)] \lambda_{i}^{g}} - \frac{(b - \theta)u_i}{\alpha} \tag{18}$$

where $\mu$ is defined below (A31); and where $m_i$ is chosen ahead of $g_i$ and $\tau_i$.

To choose $\delta$ and $\lambda^{cb}$, substitute (16)-(18) into the government loss function at (4) and dif-
ferentiate with respect to $\delta$ and $\lambda^{cb}$ to obtain first order conditions for those parameters.

There are two solutions that satisfy both first order conditions. One is $\delta = 1$, and either $\beta + \mu \gamma = 0$ (ie $\lambda_{2}^{g} = 0$, so that $\mu = -\beta / \gamma$), or $\hat{\pi}^{cb} = \hat{\pi}$. But when $0 \leq \delta < 1$ and $\beta + \mu \gamma \neq 0$, both first order conditions are also satisfied if

$$\delta = \frac{\left\{ \phi^2 \lambda^{cb} \lambda_{2}^{g} + \alpha \gamma^2 s^2 (\lambda^{cb} - \lambda_{2}^{g}) + \alpha [\alpha \gamma s]^2 + \phi^2 \lambda_{2}^{g} (\hat{\pi}^{cb} - \hat{\pi}) \right\}}{\phi^2 \lambda^{cb} \lambda_{2}^{g} + \alpha [\alpha \gamma s]^2 (\lambda^{cb} - \lambda_{2}^{g}) - \phi (\beta \phi + \gamma \lambda) \lambda_{2}^{g} \lambda_{2}^{g}} \tag{19}$$

It is easy to check that the government’s expected losses are minimised for any $(\delta, \lambda^{cb})$
pair that satisfy (19). And since any pair satisfying (19) is equally good, a fully independ-
ent central bank: $\delta = 0$ and $\lambda^{cb} = \frac{(\alpha \gamma s)^2 \lambda_{2}^{g}}{(\alpha \gamma s)^2 + \phi^2 \lambda_{2}^{g}} + \alpha (\hat{\pi} - \hat{\pi}^{cb}) > 0$ would be an optimal so-
lution. But, whatever the $\delta$ value, the overall outcome is no better than the simultane-
ous moves case whether the central bank chooses its own $\hat{\pi}^{cb}$ or not [see (A35)]. And for
every $\delta$ value, it leads to negative tax revenues on average as long as $\lambda^{cb} \leq \lambda_{2}^{g}$; or if $\hat{\pi}^{cb}$
lies sufficiently far below $\hat{\pi}$.

These tax results follow from (18), where we continue to assume $\hat{\pi}^{cb} < \hat{\pi}$. So discretion-
ary tax revenues are sure to be net negative (that is, tax revenues will be below the budget
balancing trend in total) if $\lambda^{cb} \leq \lambda^g$, but could turn positive if a large enough value of $\lambda^{cb} > \lambda^g$ were imposed; a feature that could be exploited under instrument independence. But would any rational government ever do so? We deal with the danger of an override or a government bailout in section 8.

**Target independence.** In this leadership regime, stage 1 requires $\delta$ and $\lambda^{cb}$ to be chosen from $\partial EL^g / \partial \delta = 0$ and $\partial EL^{cb} / \partial \lambda^{cb} = 0$ respectively. The former is the same as in the instrument independence case; the latter is obtained by substituting (16)-(18) into (5).

There are two solutions to this pair of first order conditions. When $\delta = 1$, and $\beta + \mu \gamma = 0$ or $\hat{\pi}^{cb} = \hat{\pi}$, the central bank is indifferent to $\lambda^{cb}$. But if $\delta \neq 1$, the optimal value of $\lambda^{cb}$ from the central bank’s perspective is:

$$\lambda^{cb} = -\delta / (1 - \delta).$$

Hence, with the permissible values of $\delta$ constrained to the interval $[0, 1]$, $\delta = 0$ is the only value for which $\lambda^{cb}$ is non-negative.

Unfortunately it may not be clear which of the two solutions will be chosen in this case. When the government/society chooses $\delta = 1$, we get positive values for both $EL^g$ and $EL^{cb}$ irrespective of the value of $\lambda^{cb}$ -- see equations (A38) and (A39). But if the government and central bank pick $\delta = 0$ and $\lambda^{cb} = 0$ respectively, we get $EL^g$ positive and $EL^{cb} = 0$. The trouble is there is no guarantee that the second solution for $EL^g$ is less than (A38), although it is certain to be smaller if $\hat{\pi}^{cb} \to \hat{\pi}$ and $\lambda^g > (\alpha \gamma s)^2 / (2 \phi^2)$. In other words, the monetary authorities can only guarantee to get to their preferred solution if the government has a commitment to fiscal sustainability, *and* if the central bank is prepared to set its own inflation target close enough to the government’s target not to trigger an override back to $\delta = 1$. If both players learn that these conditions in their own best interests, then both the bank and government will get their most preferred outcomes. This solution then generates inflation at the central bank’s preferred target, and net tax revenues that remain negative on average.
7. A Summary of the Results.

The inflation, output and net tax revenue outcomes of all seven policy regimes considered so far are collected together in table 1. They all follow by inserting the optimal values for $\delta$ and $\lambda^{cb}$ derived for each regime, into the stage 2 equations for inflation, output and net revenues under monetary leadership, fiscal leadership, or simultaneous decision making.

| Table 1: Inflation and Net Tax Revenue Outcomes of Different Policy Interactions |
|---------------------------------|-----------------|-----------------|
|                                 | $E(\pi^*)$      | $E(y^*)$        | $E(\tau^*)$ |
| Money leads I                   | $\hat{\pi}^{cb} + \phi \lambda^{cb} / \alpha$ | 0               | $\frac{\gamma s[\alpha (\hat{\pi}^{cb} - \hat{\pi}) + \lambda^{cb} - \lambda^g_1]}{\phi \lambda^{eg}_2} < 0$ |
| (instr. indep.)                 |                 |                 |              |
| Money leads II                  | $\hat{\pi}^{cb}$ | 0               | $\frac{\gamma s[\alpha (\hat{\pi}^{cb} - \hat{\pi}) - \lambda^g_1]}{\phi \lambda^{eg}_2} < 0$ |
| (target indep.)                 |                 |                 |              |
| Fiscal leads I                  | $\hat{\pi}$     | 0               | 0            |
| (instr. indep.)                 |                 |                 |              |
| Fiscal leads II                 | $\hat{\pi}$     | 0               | 0            |
| (Bank of England)               |                 |                 |              |
| Fiscal leads III                | $\hat{\pi}^{cb} < E(\pi^*) < \hat{\pi}$ | 0               | $E(\tau^*) < 0$ |
| (target indep.)                 |                 |                 |              |
| Simultaneous I                  | $\hat{\pi} + \lambda^{cb} / \alpha$ | 0               | $\frac{\gamma s[\lambda^{cb} - \lambda^g_1]}{\phi \lambda^{eg}_2} < 0$ |
| (instr. indep.)                 |                 |                 |              |
| Simultaneous II                 | $\hat{\pi}^{cb}$ | 0               | $\frac{\gamma s[\alpha (\hat{\pi}^{cb} - \hat{\pi}) - \lambda^g_1]}{\phi \lambda^{eg}_2} < 0$ |
| (target indep.)                 |                 |                 |              |

**Notes:**

a) $\lambda^{cb}$ in “Money leads I” is the optimal value below (25), satisfying (24) when $\delta = 0$. b) $\lambda^{cb}$ in “Simultaneous I” is the optimal value from (10) when $\delta = 0$. c) The solution for “Simultaneous II” assumes $\hat{\pi}^{cb} < \hat{\pi}$ by an amount small enough not to trigger a return to a dependent central bank (see section 5.1). d) The inflation inequalities in “Fiscal leads III” assume $\delta$ to be small: $\delta < \beta (\phi - \eta \Lambda) \{\lambda (\hat{\pi}^{cb} - \hat{\pi})\} / \{\lambda \phi \gamma \Lambda \lambda^g_1\}$. e) These results assume $\lambda^{cb} < \lambda^g_1$, which is the case for all solutions in this table.
Conclusions on the need for fiscal constraints:

a) Most important, all regimes have negative net tax revenues on average: \( E(\tau^*) < 0 \). The only exceptions are the fiscal leadership regime with instrument independence, or with extended instrument independence as in the Bank of England version.

b) Since \( E\tau_i^* < 0 \) in all but these two cases, the other institutional structures will always end up increasing debt. Any limits set for the debt ratio will be exceeded eventually. And since revenues are lower, budget deficits will be larger in the absence of fiscal leadership.

c) The threat to sustainability may not be obvious to the policy maker since those deficits could be quite small depending on the fiscal multipliers, sacrifice ratio \((1/\alpha)\), the savings rate, the commitment to sustainability, the priority for stabilisation, and the difference in inflation targets. Hence, a rule which restricts only the size of allowable deficit may not prevent the steady increase in debt that threatens long run sustainability.

d) This is why both monetary leadership and simultaneous decision making regimes need to have fiscal constraints imposed upon them if they are to be sustainable. And why those constraints have to be formulated as debt limits, not deficit limits.

e) Fiscal leadership eliminates the inflationary bias of the other regimes, and results in balanced budgets (on average) without loss in expected output or output volatility. That implies fewer expansionary budgets and more effective expenditure controls. The ECB should find this a much more comfortable environment to operate in.

f) These results hold independently of the commitment to the debt rule \((\lambda_2^g)\), or its target value \((\theta)\); and independently of the government’s preference to stabilise or spend \((\lambda_1^g, s)\); and of the economy’s transmission parameters \((\alpha, \beta, \gamma)\).

Results on which regimes are feasible:

In practice, not all these regimes are feasible. The target independence versions typically produce outcomes which are inferior for the government, even if they deliver better results in terms of the central bank objectives and preferences. They would therefore be blocked by the government which still has the final say on the choice of regime through its initial choice of \(\delta\). If it finds that it can expect better outcomes on average in \(EL^a\) by reverting to instrument independence, then it will do so irrespective of any benefits of target independence that may appear for the central bank in terms of \(EL^b\).
By inspection in table 1, this happens for sure with fiscal leadership. It also happens in simultaneous decision making if $\lambda^F_i \leq 1$ (and possibly larger values). And it happens under monetary leadership unless $\alpha > 1$ (which, as the inverse of the sacrifice ratio, will not be the case). We can therefore rule those regimes out as being those which the government, in its own electoral interests, would never actually choose when designing the general policy framework. Appendix B derives the exact conditions under which target independence will be blocked in each policy framework.

**Results from comparisons between regimes:**

From Table 1, it is clear that the government’s objectives are better served by fiscal leadership under instrument independence. The central bank however may be worse off than in the other solutions, depending on how much $\hat{\pi}$ and $\hat{\pi}^{cb}$ differ. However, the simultaneous moves regime approaches fiscal leadership as the debt rule becomes “harder”: $EL^z \rightarrow 0, \lambda^{cb} \rightarrow 0, \pi^* \rightarrow \hat{\pi}$, and $E\tau^*_t \rightarrow 0$ as $\lambda^F_2 \rightarrow \infty$; as does monetary leadership (inflation outcomes excepted).

8. Extensions: Risk, Uncertainty and Structural Reform

8.1 The Danger of a Government Override

From Table 1, it is obvious that negative tax revenues could be overcome in the Monetary Leadership and Simultaneous regimes if governments could impose a large enough value of $\lambda^{cb}$ as part of its instrument independence package: $\lambda^{cb} > \lambda^F_i + \alpha(\hat{\pi} - \hat{\pi}^{cb})/(1 - \delta)$ and $\lambda^{cb} > \lambda^F_1$ would be sufficient in each case. But this has to be done without reducing $\delta$ via (19) or (10) at the same time, and that will imply a loss in performance. This loss in performance arises because there will be a disproportionate increase in inflation if $\lambda^{cb}$ increases but $\delta$ is not decreased as the optimality conditions require, and (6) or (16) show. Similarly any increases in $E(\tau^a)$ will fall short of what is needed. The point here is that any government under budgetary pressure may itself reserve the right to override the central bank’s monetary policy. But it cannot do so without cost. Fiscal leadership however, is protected from an override from a government in trouble since the outcomes from that
regime are independent of $\lambda^b, \delta, \pi$ or any other parameters which the government could influence. This demonstrates that fiscal pre-commitment is the key element in our results.

8.2 Structural Reforms, Uncertainty and Volatility in the Tax Revenues

Are there other risks in the form of uncertainty or volatility in the tax revenue streams that would cause us to choose one regime over another? This question is easy to answer. From Table 1 we know that fiscal leadership, in versions I and II, is superior to any other regime with the same value of $\pi$. This holds irrespective of the parameters in the model; of the institutional and preference parameters; or of the uncertainty in those parameters. So whether they change because they are genuinely uncertain, or because they are poorly estimated, or because of a programme of reforms designed to increase competition in the markets, in wage setting, or to reduce tax distortions, fiscal leadership will continue to dominate. All that can happen is that the degree of relative superiority may increase or decrease, or the variability of the outcomes increase/decrease.

Second, uncertainty in the form of demand shocks, $\varepsilon_i$, has had no affect on the choice of regime or its outcomes. But supply shocks do affect the variability of output and net tax revenues. However these effects are the same in each regime, so the ranking between them remains unchanged. For example we have,

$$Var(y^*_i) = \sigma^2 / \alpha^2$$

and

$$Var(\tau^*) = (b - \theta)^2 \sigma^2 / \alpha^2$$

in each case, where $Var(u_i) = \sigma^2$ and $b > \theta$. As a result

$$\partial Var(y^*_i) / \partial \alpha = -2\sigma^2 / \alpha^3 < 0$$

$$\partial Var(\tau^*) / \partial \alpha = -2(b - \theta)^2 \sigma^2 / \alpha^3 < 0$$

and

$$\partial Var(\tau^*) / \partial \theta = -2(b - \theta)\sigma^2 / \alpha^2 < 0.$$

That means both a flatter Phillips curve (lower $\alpha$), as might happen with reforms designed to reduce taxation or deregulate labour markets, and a decrease in $\theta$, will increase the uncertainty in output and tax revenues. So more ambitious fiscal targets, more ambitious social targets, and more flexible labour markets will mean more variable tax revenues, and more unstable budgets. In addition, the worse the inherited position ($b$), the stronger is that effect. Similarly, any increase in the variability of supply side shocks (shortages in oil or raw materials, or financial stress, or simply the effects of increasing global competition) will increase the uncertainty in output and tax revenues. But on the positive side,
the volatility of the budget is less than the volatility of output; and increasing flexibility in any of the other parameters does not play a role in budget uncertainty.

8.3 Market Reforms, Globalisation and Financial Integration

We now consider the effects of changes in the parameters $\alpha$ and $s$ on the outcomes, rather than on the volatility of our different policy regimes. In particular, do reforms or external changes that affect those two parameters make monetary leadership more attractive as a policy regime? We can interpret a fall in $\alpha$ in a number of different ways. It could be the result of transnational wage bargaining, or the effect of locational competition and globalization on the slope and position of the Phillips curve (Demertzis and Hughes Hallett 1998; Bean 2006; Pain et al, 2006). Or it could be the result of reducing business taxes; or wage taxes if the price margins of imperfectly competitive firms are sufficiently sensitive (Bokan and Hughes Hallett, 2007). By contrast, reducing wage bargaining power, or employment protection, or hiring and firing costs, will have little effect on the slope as opposed to the position of the Phillips curve. In fact increasing the degree of internal competition, or deregulating labour markets would eventually increase $\alpha$.

Equally we can interpret falls in $s$ to be the result of increased risk sharing in a currency zone that follows from the greater consumption smoothing possibilities. And if consumption risks are reduced, then perhaps we should see a fall in $\lambda_2^s$ as well – a falling concern for the distribution of income. But if tax competition is rising, then $\lambda_2^s$ might rise because the low income/low skill population is at an increased risk. A more important point is that worsening demographics, implying lower pensions and greater health care costs, and possibly higher taxes in the future, would all tend to raise $s$ – as has happened in Japan for example. Fear of globalization would have the same effect.

The impact of changes in $\alpha$ and $s$ are shown in tables 2 and 3. The results are mixed. Fiscal leadership (in either form) is not affected of course; its superior outcomes remain.

---

17 Razin and Binyamini (2007) argue instead that it is the result of trade, greater competition and migration as markets integrate. But it could equally well be the result of reduced market frictions (Smets and Wouters 2007) or distortions as argued here; or of greater credibility and effectiveness in monetary policy (Roberts 2006, Boivin and Giannoni 2006), especially when it comes to anchoring expectations (Williams 2006).

18 Bokan and Hughes Hallett (2007) show how these different reforms change the model parameters.
Under monetary leadership, average inflation would almost certainly fall with locational competition, more flexible markets, lower taxes (lower $\alpha$ values); because $(\alpha\gamma)^2$ will be small relative to $\phi^2 \lambda^g$ unless $\lambda^g \rightarrow 0$ [and since $1 - \gamma \delta s > 0$ unless $\gamma$ is implausibly large]. But inflation will rise with greater competition within the markets, or if there is no commitment to sustainable fiscal policies or social equity. Thus structural reforms that reduce $\alpha$ might make the inflation target easier to reach (because the loss in output gap is smaller), but at the cost of more volatile output and tax revenues. Similarly flattening $\alpha$ will lower net tax revenues; that is, reduce them on average as well as destabilizing them.

Switching to simultaneous decision making produces the same conclusions under exactly the same conditions. The lesson here is that increasing market flexibility or reducing tax distortions on their own increase the lead of fiscal leadership; while stronger competition policy would reduce it.

Increased savings rates would also not affect the overall superiority of fiscal leadership. But they would reduce average inflation under monetary leadership, unless $\hat{\alpha} \rightarrow \hat{\pi}$ and $\theta$ is small or $\lambda^g$ large. And they make net tax revenues more negative unless $\lambda^g = 0$. Thus higher savings (despite being used to strengthen fiscal policies; or to support social equity, pensions or other public spending)\(^{19}\) typically reduce inflationary pressures. Under simultaneous decisions, increased saving would lead to larger tax revenues and higher average inflation because fiscal policy can be used more effectively to support output. The lesson here is that higher savings rates will improve monetary leadership (relative to fiscal leadership) provided there is some fiscal commitment, but have ambiguous effects under simultaneous decisions. Hence the effects of ageing will make monetary leadership look better, whereas greater consumption smoothing would make it look worse.

Finally, we know that raising $\lambda^g$ closes the gap between fiscal leadership and the other regimes. So an increasing commitment to fiscal discipline or social equity, as might happen if protecting low skill jobs from global competition is important, reduces the advantage of fiscal leadership; but increases it again when income equality improves.

\(^{19}\) By section 6.2 these savings may be both public and private, even if they are used for fiscal purposes.
Table 2: Sensitivity of the Outcomes to Changes in the Inflation-Output Gap Trade-off

<table>
<thead>
<tr>
<th></th>
<th>( \partial E(\pi^*) / \partial \alpha )</th>
<th>( \partial E(\gamma^*) / \partial \alpha )</th>
<th>( \partial E(\tau^*) / \partial \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money leads I (instr. indep.)</td>
<td>((1 - \gamma \delta_k)\gamma^2 s^2 \frac{2}{\lambda_2^g} \left[ \phi^2 \lambda_2^g \gamma^2 s^2 + \phi^2 \lambda_2^g \right] + \beta(\hat{\pi} - \hat{\pi}^{ch}) )</td>
<td>0</td>
<td>( \frac{\gamma s}{\phi^2 \lambda_2^g} \left[ 2\alpha \gamma^2 s^2 \phi^2 \lambda_2^g \lambda_2^g (1 - \gamma \delta_k) \right] + \frac{\beta \phi^2 \lambda_2^g}{\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g} )</td>
</tr>
<tr>
<td>Fiscal leads I (instr. indep.)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fiscal leads II: Bank of England variant</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Simultaneous I (instr. indep.)</td>
<td>( \gamma^2 s^2 \lambda_2^g \left[ \phi^2 \lambda_2^g (\phi - 2\alpha \beta) - \alpha^2 \gamma^2 s^2 \right] )</td>
<td>0</td>
<td>( \frac{\gamma s \lambda_2^g \left[ \phi \lambda \alpha / \partial \alpha - (\lambda - \lambda_2^g) \beta \right]}{(\phi \lambda_2^g)^2} )</td>
</tr>
</tbody>
</table>

Table 3: Sensitivity of the Outcomes to Changes in the Savings Ratio “s”

<table>
<thead>
<tr>
<th></th>
<th>( \partial E(\pi^*) / \partial s )</th>
<th>( \partial E(\gamma^*) / \partial s )</th>
<th>( \partial E(\tau^*) / \partial s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money leads I (instr. indep.)</td>
<td>( \alpha \gamma^2 s \lambda_2^g \left[ 2\phi^2 \lambda_2^g (1 + \alpha \beta) - \gamma \delta_k \right. ) \left. \right] \left[ (\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g) \right] + \gamma \theta(\hat{\pi} - \hat{\pi}^{ch}) )</td>
<td>0</td>
<td>( -\frac{\gamma \lambda_2^g (\phi + \gamma \delta_k) \left[ \phi^2 \lambda_2^g - \alpha^2 \gamma^2 s^2 \right]}{(\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g)^2} )</td>
</tr>
<tr>
<td>Fiscal leads I (instr. indep.)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fiscal leads II: Bank of England variant</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Simultaneous I (instr. indep.)</td>
<td>( 2\alpha \gamma^2 s \phi \lambda_2^g \lambda_2^g (1 + \alpha \beta) \left[ (\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g) \right] )</td>
<td>0</td>
<td>( -\gamma(1 + \alpha \beta) \lambda_2^g \left[ \frac{\alpha^2 \gamma^2 s^2 - \phi^2 \lambda_2^g}{(\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g)^2} \right] )</td>
</tr>
</tbody>
</table>
9 Conclusions:

a) Fiscal leadership with instrument independence and an independent central bank provides a superior performance compared to other institutional arrangements that account for the interactions between fiscal and monetary policies.

b) Only fiscal leadership, under the same conditions, will ensure sustainable fiscal policies and sustainable public finances. Other regimes will need a fiscal constitution to restrain those policies.

c) These conclusions are robust to the behavioural changes that may emerge from globalization, demographic shifts, and changes in the mix of demand and supply shocks.

d) The advantages of fiscal leadership would be jeopardized if the central bank were given leadership or target independence in addition to instrument independence – which means that enforcing fiscal restrictions such as the Stability and Growth Pact would become more difficult and Sarkozy-like economic populism is bound to emerge at some point.

e) These results show that the natural restraint on government spending under independent monetary policies, where tax revenues reduce the savings needed to fund those expenditures, is not strong enough by itself to discipline fiscal policy makers – unless accompanied by the threat of monetary discipline and a long term fiscal target. In other words, there must be a no bail out mechanism to prevent expansions which push costs on others. This is important because the major challenges for the single currency evidently come from fiscal pressure, particular under conditions of slow growth, ageing populations, the pressures of globalization, and financial instability.
References


European Commission (EC, 2002)”Public Finances in EMU:2” European Economy: Studies and Reports, 4, European Commission, Brussels.


Surveillance in Real Time”, Discussion Paper 124, de Nederlandsche Bank (Dutch Central Bank), Amsterdam.
APPENDIX A: SOLVING THE MODEL

Given one of the two interpretations in section 5.2, we can solve for $\pi^*_i, \pi_i$ and $y_i$ from (1) and (2) to obtain the following reduced forms:

$$\pi_i(g_i,m_i) = (1 + \alpha \beta)^{-1}[\alpha \beta m_i + \alpha \gamma g_i + m_i^\epsilon + \frac{\gamma}{\beta} g_i^\epsilon + \alpha \epsilon_i + u_i]$$  \hspace{1cm} (A1)

$$y_i(g_i,m_i) = (1 + \alpha \beta)^{-1}[\beta m_i + \gamma g_i - \beta m_i^\epsilon - \gamma g_i^\epsilon + \epsilon_i - \beta u_i].$$  \hspace{1cm} (A2)

Solving for $\tau_i$ using (3) and (A2), then yields

$$\tau_i(g_i,m_i) = [s(1 + \alpha \beta)]^{-1}[(1 + \alpha \beta + sb \beta)m_i - (1 + \alpha \beta - sb \gamma)g_i$$

$$- sb \beta m_i^\epsilon - sb \gamma g_i^\epsilon + sb(\epsilon_i - \beta u_i)].$$  \hspace{1cm} (A3)

This version of the model can now be substituted into the policy makers’ objectives given by (4) or (5). Optimisation according to the regime in question then provides optimal reaction functions for $m_i$ and $g_i$. From there we get expressions for the expected values of $m_i$ and $g_i$; and substituting them into (A1)-(A3) gives the outcomes for each regime.

APPENDIX B: OUTCOMES UNDER THE DIFFERENT REGIMES

1) The Simultaneous Moves Regime: with instrument independence

In this regime, the government first chooses the parameters $\delta$ and $\lambda^{cb}$. Then, in a second stage, the government and monetary authority set their policy instruments, $g_i$ and $\tau_i$, for the government and $m_i$ for the central bank, in a Nash equilibrium given the values of $\delta$ and $\lambda^{cb}$ just chosen and a common inflation target $\hat{\pi}$. Private agents understand the game and form rational expectations about future prices and policies before the second stage:
that is, before the policy makers implement their policies, but after the institutional parameters $\delta$ and $\lambda^{cb}$ have been set. The policy game therefore takes the form:

**Stage 1**

The government solves the problem:

$$
\min_{\delta, \lambda^c} EL^g (g_i, m_i, \delta, \lambda^{cb}) = E \left\{ \frac{1}{2} \left[ \pi_i (g_i, m_i) - \hat{\pi} \right]^2 - \lambda_i^c [y_i (g_i, m_i)] \right\} \\
+ \frac{\lambda^g}{2} E [(b - \theta) y_i (g_i, m_i) - \tau_i (g_i, m_i)]^2 
$$

(A4)

where $L^g (g_i, m_i, \delta, \lambda^{cb})$ is (4) evaluated at $(g_i, m_i, \delta, \lambda^{cb})$, and $E$ denotes expectations.

**Stage 2**

1. Private agents form rational expectations about future prices $\pi^f_i$ from (A1), before the shocks $u_i$ and $\varepsilon_i$ are realized.
2. The shocks $u_i$ and $\varepsilon_i$ are realized and observed by the government and by the central bank.
3. The government chooses $g_i$, taking $m_i$ as given, to minimize $L^g (g_i, m_i, \delta, \lambda^{cb})$ where $\delta$ and $\lambda^{cb}$ denote the values determined at stage 1.
4. At the same time, the central bank chooses $m_i$, taking $g_i$ as given, to minimize

$$
L^{cb} (g_i, m_i, \delta, \lambda^{cb}) = \frac{(1 - \delta)}{2} \left[ \pi_i (g_i, m_i) - \hat{\pi} \right]^2 - (1 - \delta) \lambda^{cb} \left[ y_i (g_i, m_i) \right] \\
+ \delta L^g (g_i, m_i, \delta, \lambda^{cb}) 
$$

(A5)

We solve this game by solving backwards: for the policy choices (stage 2) first; and then substituting the results back into (A4) to determine the optimal institutional parameters $\delta$ and $\lambda^{cb}$. From the Nash equilibrium at stage 2, we get

$$
\pi_i (\delta, \lambda^{cb}) = \hat{\pi} + \frac{(1 - \delta) \beta \phi \lambda^{cb} + \delta (\beta \phi + \gamma \Lambda) \lambda^g}{\alpha [\beta \phi + \delta \gamma \Lambda]} 
$$

(A6)

$$
y_i (\delta, \lambda^{cb}) = -u_i / \alpha , 
$$

(A7)

and

$$
\tau_i (\delta, \lambda^{cb}) = \frac{(1 - \delta) \beta \phi (\lambda^{cb} - \lambda^g)}{[\beta \phi + \delta \gamma \Lambda] \lambda^g} - \frac{(b - \theta) u_i}{\alpha} 
$$

(A8)

where $\phi = 1 + \alpha \beta - \gamma \delta k$ ,

(A9)

and

$$
\Lambda = 1 + \alpha \beta + \beta \delta k . 
$$

(A10)
Evidently $\Lambda$ is positive. We assume $\phi$ to be positive as well. One should expect $\phi > 0$ since, with $0 < 1$ and $s < 1$, fiscal policy would otherwise have to have such a strong impact on national income that, together with a Phillips curve that is sufficiently flat and weak monetary transmissions, government expenditures would be able to simultaneously boost output and be transferred to pay debt down without worsening the budget or debt ratios at the same time. That would require a fiscal policy multiplier of $\gamma > (1 + \alpha \beta)/(1 - s)$. In fact, with $s \approx 0.5$ (section 5.2) and debt target of 50% of GDP, it would require fiscal multipliers in excess of 4 or 5. This is hardly plausible; and numerical evaluations for ten of the larger OECD economies place $\phi$ close to unity. Nevertheless, in order to get $\phi < 0$, output would have to be capable of growing fast enough to generate sufficient revenues to boost output when needed and to pay down the debt. If that is not possible, then one must to come at the expense of the other.

Substituting (A6)-(A8) back into (A4), we now get the stage 1 solution from:

$$
\min_{\delta, \lambda^{ch}} EL^g (\delta, \lambda^{ch}) = \frac{1}{2} \left\{ \frac{(1 - \delta) \beta \phi \lambda^{cb} + \delta (1 + \alpha \beta) (\beta + \gamma) \lambda^g}{\alpha [\beta \phi + \delta \gamma \Lambda]} \right\}^2
+ \frac{\lambda^g}{2} \left\{ \frac{(1 - \delta) \beta \gamma \Lambda (\lambda^{cb} - \lambda^g)}{[\beta \phi + \delta \gamma \Lambda] \lambda^g} \right\}^2.
$$

(A11)

Since $\beta \phi + \delta \gamma \Lambda \neq 0$, this part of the problem has two first order conditions:

$$
(1 - \delta) \phi \lambda^g \left( (1 - \delta) \beta \phi \lambda^{cb} + \delta (\beta \phi + \gamma \Lambda) \lambda^g \right)
- (1 - \delta)^2 \gamma^2 \alpha^2 s^2 \beta (\lambda^g - \lambda^{cb}) = 0
$$

(A12)

and

$$
\lambda^g \phi \left( (1 - \delta) \beta (\phi - \eta \Lambda) \lambda^{cb} + \delta (\beta \phi + \gamma \Lambda) \lambda^g \right)
- (1 - \delta)^2 \gamma^2 \alpha^2 s^2 \beta (\lambda^g - \lambda^{cb})^2 = 0.
$$

(A13)

There are two sets of values for $\delta$ and $\lambda^{cb}$ which satisfy this pair of first-order conditions. The first solution is $\delta = 1$ and $\lambda^{cb} = \lambda^g$. That solution describes an entirely dependent central bank. However, it is inferior to the second solution where $0 \leq \delta < 1, \lambda^{cb} \neq \lambda^g$ and
\[ \delta = \frac{\beta \phi^2 \lambda^{ch} \lambda_2^g + \alpha^2 \gamma^2 s^2 \beta (\lambda^{ch} - \lambda_1^g)}{\beta \phi^2 \lambda^{ch} \lambda_2^g + \alpha^2 \gamma^2 s^2 \beta (\lambda^{ch} - \lambda_1^g) - \phi (\beta \phi + \gamma \Lambda) \lambda_2^g} \]  

(A14)

Out of these two possibilities, we can check which yields the lowest welfare loss by evaluating (A11) under the two alternative solutions. Substituting \( \delta = 1 \) and \( \lambda^{ch} = \lambda_1^g \) into (A11) results in

\[ EL^g = \frac{(\lambda_1^g)^2}{2\alpha^2} \]  

(A15)

But substituting (A14) into the right-hand-side of (A11) yields

\[ EL^g = \frac{(\lambda_1^g)^2}{2\alpha^2} \left( \frac{\alpha^2 \gamma^2 s^2}{\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g} \right) \]  

(A16)

For positive values of \( \lambda_2^g \) the value of (A15) is greater than (A16).

**Target independence**: Here the central bank chooses its own inflation target and priorities, \( \hat{\pi}^{ch} \) and \( \lambda^{ch} \). It produces a different distribution of outcomes, but an inferior welfare result because the government retains the right to manipulate the value of \( \delta \). The only difference is that the terms

\[ (1 - \delta) \beta \phi \hat{\pi}^{ch} + \delta (\beta \phi + \gamma \Lambda) \hat{\pi} \]  

and

\[ \frac{\alpha \beta \gamma s (1 - \delta) (\hat{\pi}^{ch} - \hat{\pi})}{\beta \phi + \delta \gamma \Lambda \lambda_2^g} \]  

(A17)

need to be added into (6) and (8) respectively, while (7) remains unchanged. It is easy to check that this solution produces lower inflation, but reduces tax revenues when \( \hat{\pi}^{ch} < \hat{\pi} \). The inferior welfare outcome then follows because the central bank will always choose a value of \( \hat{\pi}^{ch} \) lower than \( \hat{\pi} \), but sufficiently close to it, to make the government to choose \( \delta = 0 \). That minimizes the bank’s loss function, but leaves the government/society’s losses somewhere between (A15) and (A16).²⁰

2) **Fiscal Leadership: with instrument independence.**

In this regime, the government still chooses the institutional parameters \( \hat{\delta} \) and \( \lambda^{ch} \). But the second stage is a Stackelberg game in which fiscal policy takes on a leadership role. In

²⁰ Hughes Hallett and Weymark (2004b), section 4.4, gives the exact calculations.
that stage, the government and the monetary authority set their policy instruments, given \( \delta \) and \( \lambda^b \) values determined knowing that fiscal leadership will follow. Put formally:

**Stage 1**

The government solves the problem:

\[
\min_{\delta, \lambda^b} EL^g (g_t, m_t, \delta, \lambda^b) = E \left\{ \frac{1}{2} [\pi_t (g_t, m_t) - \hat{\pi}]^2 - \lambda^g_t [y_t (g_t, m_t)] \right\}
\]

\[
+ \frac{\lambda^g_t}{2} E [(b - \theta) y_t (g_t, m_t) - \tau_t (g_t, m_t)]^2
\]

(A18)

where \( L^g (g_t, m_t, \delta, \lambda^b) \) is (4) evaluated at \( (g_t, m_t, \delta, \lambda^b) \), and \( E \) denotes expectations.

**Stage 2**

5. Private agents form rational expectations about future prices \( \pi_t^e \) before the shocks \( u_t \) and \( \varepsilon_t \) are realized.

6. The shocks \( u_t \) and \( \varepsilon_t \) are realized and observed by the government and by the central bank.

7. The government chooses \( g_t \), before \( m_t \) is chosen by the central bank, to minimize \( L^g (g_t, m_t, \bar{\delta}, \bar{\lambda}^b) \) where \( \bar{\delta} \) and \( \bar{\lambda}^b \) are at the values determined at stage 1.

8. The central bank then chooses \( m_t \), taking \( g_t \) as given, to minimize

\[
L^b (g_t, m_t, \bar{\delta}, \bar{\lambda}^b) = \frac{(1 - \bar{\delta})}{2} [\pi_t (g_t, m_t) - \hat{\pi}]^2 - (1 - \bar{\delta}) \bar{\lambda}^b [y_t (g_t, m_t)]
\]

\[
+ \bar{\delta} L^g (g_t, m_t, \bar{\delta}, \bar{\lambda}^b)
\]

We solve this game backwards: for the policy choices (stage 2) first; and then substituting the results back into (A18) to determine the optimal institutional parameters \( \delta \) and \( \lambda^b \).

From stage 2, we get

\[
\pi_t (\delta, \lambda^b) = \hat{\pi} + \frac{(1 - \delta) \beta (\phi - \eta \Lambda) \lambda^b + \delta (\beta \phi + \gamma \Lambda) \lambda^g}{\alpha [\beta (\phi - \eta \Lambda) + \delta \Lambda (\beta \eta + \gamma)]}
\]

(A19)

\[
y_t (\delta, \lambda^b) = -u_t / \alpha ,
\]

(A20)

\[
\tau_t (\delta, \lambda^b) = \frac{(1 - \delta) \beta s (\beta \eta + \gamma) (\lambda^b - \lambda^g)}{[\beta (\phi - \eta \Lambda) + \delta \Lambda (\beta \eta + \gamma)]} \frac{(b - \theta) u_t}{\alpha}
\]

(A21)

where

\[
\eta = \frac{\partial m_t}{\partial g_t} = - \frac{\alpha^2 \gamma \beta s \lambda^g}{(\alpha \beta s)^2 + \delta \Lambda^2 \lambda^b},
\]

(A22)

with \( \phi = 1 + \alpha \beta - \gamma \theta s \) and \( \Lambda = 1 + \alpha \beta + \beta \theta s \) as before.

Substituting (A19)-(A21) back into (A17), we can now get the stage 1 solution from:
This part of the problem has two first order conditions:

\[
(1 - \delta)(\phi - \eta \Lambda) \lambda^g - (1 - \delta) \beta(\phi - \eta \Lambda) \lambda^{cb} + \delta(\beta \phi + \gamma \Lambda) \lambda^g = 0 \tag{A24}
\]

and

\[
(1 - \delta)(\phi - \eta \Lambda) \lambda^g - (1 - \delta) \beta(\phi - \eta \Lambda) \lambda^{cb} + \delta(\beta \phi + \gamma \Lambda) \lambda^g = 0 \tag{A25}
\]

where \( \Omega = \partial \eta / \partial \delta \). There are two real-valued solutions which satisfy this pair of first-order conditions.\(^{21}\) Both are satisfied when \( \delta = 1 \) and \( \lambda^{cb} = \lambda^g \). That solution describes a fully dependent central bank. However, it is inferior to the second solution: \( \delta = \lambda^{cb} = 0 \). In that solution, the central bank is fully independent and exclusively concerned with the economy’s inflation performance.

Out of these two possibilities, the solution which yields the lowest welfare loss, as measured by the government’s (society’s) loss function, can be identified by comparing (A23) to the losses to be expected under the alternative arrangement. Substituting \( \delta = 1 \) and \( \lambda^{cb} = \lambda^g \) into (A23) results in

\[
EL^g = \frac{(\lambda^g)^2}{2 \alpha^2} \tag{A26}
\]

But substituting \( \delta = \lambda^{cb} = 0 \) into the right-hand-side of (A23) yields

\[
EL^g = 0. \tag{A27}
\]

Consequently, under fiscal leadership, society’s welfare losses are minimized when the government appoints independent central bankers who are concerned only with the achievement of a mandated inflation target and disregard their on output.

---

\(^{21}\) Because \( \eta \) is a function of \( \delta \), (A25) is quartic in \( \delta \). This polynomial has four distinct roots, of which only two are real-valued. For the complete solution, see Hughes Hallett and Weymark (2004b). Note that \( \beta \phi + \gamma \Lambda, \phi - \eta \Lambda, \Omega \), and \( \beta \eta + \gamma \) are all positive if \( \phi > 0 \), with the exception of \( \beta \eta + \gamma \) which is zero when \( \delta = 0 \).
The Bank of England variant: We can analyse this regime using (A19)-(A21) since the second stage of the game as a function of $\delta$ and $\lambda^{cb}$ is unchanged. Moreover, since the government retains the right to choose $\delta$, the first order conditions for the first stage are (A25) for $\partial EL^g / \partial \delta = 0$; and $\partial EL^{cb} / \partial \lambda^{cb} = 0$ can be obtained from

$$EL^{cb} = \frac{1}{2} \left\{ \frac{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb} + \delta(\beta\phi + \gamma\Lambda)\lambda^g}{\alpha[\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)]} \right\}^2 + \frac{\delta\lambda^g}{2} \left\{ \frac{(1-\delta)\beta\gamma(\beta\eta + \gamma)(\lambda^{cb} - \lambda^g)}{\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)\lambda^g} \right\}.$$ 

Hence the two first order conditions for the first stage are now (A25) and

$$\left\{ \frac{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb} + \delta(\beta\phi + \gamma\Lambda)\lambda^g}{\alpha[\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)]} \right\} \left\{ \frac{(1-\delta)\beta(\phi-\eta\Lambda)}{\alpha[\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)]} \right\} + \delta\lambda^g \left\{ \frac{(1-\delta)\beta\gamma(\beta\eta + \gamma)(\lambda^{cb} - \lambda^g)}{\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)\lambda^g} \right\} = 0.$$ 

Notice that $\delta = 1, \lambda^{cb} = \lambda^g$ is still one solution; and that $\delta = \lambda^{cb} = 0$ is still the other. That in turn implies the same outcomes as in fiscal leadership with instrument independence.

Target independence: We now allow the central bank to choose its own inflation target $\hat{\pi}^{cb}$. This case is more complicated because reworking the stage 2 outcomes, (14)-(16), shows that the expression for $\pi$ in (14) has the $\hat{\pi}$ term replaced by

$$\pi = \frac{\beta(\phi-\eta\Lambda)\hat{\pi}^{cb} + \delta\Lambda(\beta\eta + \gamma)\hat{\pi}}{\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)},$$ 

which is not independent of the choice of $\delta$. That means the expressions for $EL^g$ and $EL^{cb}$ will have the additional terms of

$$\pi - \hat{\pi} = \frac{\beta(\phi-\eta\Lambda)(\hat{\pi}^{cb} - \hat{\pi})}{\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)} \quad \text{and} \quad \pi - \hat{\pi}^{cb} = \frac{-\delta\Lambda(\beta\eta + \gamma)(\hat{\pi}^{cb} - \hat{\pi})}{\beta(\phi-\eta\Lambda) + \delta\Lambda(\beta\eta + \gamma)}$$

in their respective inflation components. From here it is straightforward, but tedious, to show that $EL^g = \frac{1}{2}(\hat{\pi}^{cb} - \hat{\pi})^2 > 0$ and $\partial EL^g / \partial \delta < 0$ at $\delta = \lambda^{cb} = 0$; and also that $EL^{cb}$ and $\partial EL^{cb} / \partial \delta$ are both zero at that point. Hence the government will certainly choose $\delta > 0$, even if $\lambda^{cb}$ remains at zero, to offset the fact that the central bank would otherwise achieve $\pi = \hat{\pi}^{cb}$, see (A29), while the government is trying to achieve a different (higher) inflation outcome. That means inflation outcomes will be higher, by (11) and (A22), and
expected revenues lower, than in the instrument independence or Bank of England variants. As a result fiscal policy will become unsustainable, and the outcomes less satisfactory for the central bank ($EL^b$ rises from its first best optimum of zero).

3) Monetary Leadership: with instrument independence

If the central bank has leadership and acts a Stackelberg leader, it will always be able to select the inflation target $\hat{x}^{cb}$ for its own decision rule. If the government retains responsibility for $\delta$ and $\lambda^{cb}$, the game then works as follows:

**Stage 1**

The government solves the problem:

$$\min_{\delta,x^g} EL^g(g_i,m_i,\delta,x^g) = E\left\{ \frac{1}{2} \left[ \pi_i(g_i,m_i) - \hat{\pi}_i \right]^2 - \lambda^{g}_i \left[ y_i(g_i,m_i) \right] \right\}$$

$$+ \frac{\lambda^{g}_i}{2} E[(b-\theta)y_i(g_i,m_i) - \tau_i(g_i,m_i)]^2$$

where $L^g(g_i,m_i,\delta,x^g)$ is (4) evaluated at $(g_i,m_i,\delta,x^{cb})$, and $E$ denotes expectations.

**Stage 2**

9. Private agents form rational expectations about future prices $\pi^e_i$ before the shocks $u_i$ and $\varepsilon_i$ are realized.

10. The shocks $u_i$ and $\varepsilon_i$ are realized and observed by the government and by the central bank.

11. The central bank chooses $m_i$, before $g_i$ is chosen by the government, to minimize $L^{cb}(g_i,m_i,\delta,\lambda^{cb})$ where $\delta$ and $\lambda^{cb}$ are at the values determined at stage 1.

12. The government chooses $g_i$, taking $m_i$ as given, to minimise $L^g(g_i,m_i,\delta,\lambda^{cb})$.

Solving backwards, the reduced form outcomes at stage 2 (as functions of $\delta$ and $\lambda^{cb}$) are:

$$\pi_i(\delta,\lambda^{cb}) = \frac{(\beta + \mu \gamma)\phi \hat{x}^{cb} + \delta \gamma (\Lambda - \mu \phi) \hat{x}_i + (1 - \delta)(\beta + \mu \gamma)^2 \lambda^{cb} + \delta(\beta \phi + \gamma \Lambda) \lambda^{g}_i}{(\beta + \mu \gamma)\phi + \delta \gamma (\Lambda - \mu \phi) + \alpha((\beta + \mu \gamma) + \delta \gamma (\Lambda - \mu \phi))}$$ (A30)

$$y_i(\delta,\lambda^{cb}) = -\frac{u_i}{\alpha}$$ (A31)

$$\tau_i(\delta,\lambda^{cb}) = \frac{\alpha s (\beta + \mu \gamma)(\hat{x}^{cb} - \hat{x}) + (1 - \delta) \gamma (\beta + \mu \gamma) s (\lambda^{cb} - \hat{\lambda}_i) - (b - \theta) u_i}{[(\beta + \mu \gamma)\phi + \delta \gamma (\Lambda - \mu \phi)] \hat{\lambda}^{g}_i}$$ (A32)

where $\mu = \frac{\partial g_i}{\partial m_i} = -\frac{\alpha^2 s^2 \gamma + \phi \Lambda \hat{\lambda}^{g}_i}{\alpha^2 \gamma^2 s^2 + \phi^2 \hat{\lambda}^{g}_i}$, and $\phi$ and $\Lambda$ are as before.
To choose \( \delta \) and \( \lambda^{cb} \), substitute (A30)-(A32) into the government loss function (4) and differentiate with respect to \( \delta \) and \( \lambda^{cb} \) to obtain first order conditions for those parameters:

\[
\phi \lambda^{cb}_1 \Gamma \Sigma \left\{ \alpha (\beta + \mu \gamma) \phi (\hat{\pi}^{cb} - \hat{\pi}) + \phi (1 - \delta) \lambda^{cb} + \delta (\beta \phi + \gamma \Lambda) \lambda^{cb}_1 \right\} \\
+ \alpha^2 \gamma^2 s^2 \Gamma^2 \Sigma \left\{ \alpha (\hat{\pi}^{cb} - \hat{\pi}) + (1 - \delta) (\lambda^{cb} - \lambda^{cb}_1) \right\} = 0 \quad (A33)
\]

and

\[
(1 - \delta) \phi \lambda^{cb}_2 \left\{ \alpha \Gamma \phi (\hat{\pi}^{cb} - \hat{\pi}) + \phi (1 - \delta) \lambda^{cb} + \delta (\beta \phi + \gamma \Lambda) \lambda^{cb}_2 \right\} \\
+ \alpha^2 \gamma^2 s^2 \Gamma (1 - \delta) \left\{ \alpha (\hat{\pi}^{cb} - \hat{\pi}) + (1 - \delta) (\lambda^{cb} - \lambda^{cb}_1) \right\} = 0 \quad (A34)
\]

respectively, where \( \Gamma = (\beta + \mu \gamma) \) and \( \Sigma = (\beta \phi + \gamma \Lambda)(\lambda^{cb}_1 - \lambda^{cb}) + \alpha \gamma (\hat{\pi} - \hat{\pi}^{cb})(\Lambda - \mu \phi) \).

Two solutions satisfy both (A33) and (A34). One is \( \delta = 1 \), and \textbf{either} \( \Gamma = 0 \) (i.e., \( \lambda^{cb}_2 = 0 \), so \( \mu = -\beta / \gamma \)); or \( \hat{\pi}^{cb} = \hat{\pi} \). But when \( 0 \leq \delta < 1 \) and \( \Gamma \neq 0 \), (A33) and (A34) are also satisfied if

\[
\delta = \frac{(\beta + \mu \gamma) \left\{ \phi^2 \lambda^{cb}_2 \gamma^2 s^2 (\lambda^{cb} - \lambda^{cb}_1) + \alpha \left\{ (\alpha \gamma s)^2 + \phi^2 \lambda^{cb}_1 \right\} (\hat{\pi}^{cb} - \hat{\pi}) \right\}}{(\beta + \mu \gamma) \left\{ \phi^2 \lambda^{cb}_2 \gamma^2 s^2 + (\alpha \gamma s)^2 (\lambda^{cb} - \lambda^{cb}_1) - \phi (\beta \phi + \gamma \Lambda) \lambda^{cb}_1 \right\}}
\]

(A35)

The government’s expected losses are now minimised for any \((\delta, \lambda^{cb})\) pair that satisfy (A34). Substituting back into (A30)-(A32) and the result in (4), yields that minimum as:

\[
EL^{cb} = \frac{(\lambda^{cb}_1)^2}{2 \alpha^2} \left\{ \frac{\alpha^2 \gamma^2 s^2}{\alpha^2 \gamma^2 s^2 + \phi^2 \lambda^{cb}_1} \right\}.
\]

(A36)

Since any pair satisfying (A35) is equally good, one solution would be a fully independent central bank: \( \delta = 0 \) and \( \lambda^{cb} = \frac{(\alpha \gamma s)^2 \lambda^{cb}_1}{(\alpha \gamma s)^2 + \phi^2 \lambda^{cb}_2} + \alpha (\hat{\pi} - \hat{\pi}^{cb}) > 0 \). But, whatever the value of \( \delta \), (A36) is no better than the simultaneous moves case whether the central bank chooses its own \( \hat{\pi}^{cb} \) or not. And it also leads to negative tax revenues on average as long as \( \lambda^{cb} \leq \lambda^{cb}_1 \); or if \( \hat{\pi}^{cb} \) lies below \( \hat{\pi} \) but sufficiently far away, \( \hat{\pi} - \hat{\pi}^{cb} \geq (1 - \delta) (\lambda^{cb} - \lambda^{cb}_1) / \alpha \).

**Target independence:** In this version, the stage 1 parameters, \( \delta \) and \( \lambda^{cb} \), must be chosen from \( \partial EL^{cb} / \partial \delta = 0 \) and \( \partial EL^{cb} / \partial \lambda^{cb} = 0 \) respectively. The former is given by (A32); and the latter by substituting (A30)-(A32) into (5). The first order condition from the latter is

\[\footnote{This will be the government’s preferred outcome, since the \( \delta=1 \) solution yields \( EL^{cb} = (\lambda^{cb}_1 / \alpha)^2 / 2 \).}
There are two possible solutions to this pair of first order conditions. One is when \( \delta = 1 \), and either \( \Gamma = 0 \) or \( \hat{\lambda}^{cb} = \hat{\lambda} \). The central bank is indifferent about \( \lambda^{cb} \) in this case. But if \( \delta \neq 1 \), the optimal value of \( \lambda^{cb} \) for the central bank is given by:

\[
\lambda^{cb*} = -\delta / (1 - \delta). \tag{A38}
\]

With the permissible range for \( \delta \) constrained to \([0, 1]\), the only value for which \( \lambda^{cb*} \) is non-negative is \( \delta = 0 \).

Unfortunately it is not always clear which of the two solutions will be chosen. When the government or society chooses \( \delta = 1 \), we obtain

\[
EL^g = \left\{ \frac{(\alpha s)^2 (\hat{\pi} - \hat{\lambda}^{cb})}{(\alpha s)^2 + \phi^2 \lambda^g_2} + \frac{\lambda^g_1}{\alpha} \right\}^2 + \frac{\lambda^g_2}{2} \left\{ \frac{\alpha s \phi \lambda^g_2 (\hat{\pi} - \hat{\lambda}^{cb})}{(\alpha s)^2 + \phi^2 \lambda^g_2} \right\}^2 > 0 \tag{A39}
\]

and

\[
EL^{cb} = \frac{1}{2} \left\{ \frac{(\alpha s)^2 (\hat{\pi} - \hat{\lambda}^{cb})}{(\alpha s)^2 + \phi^2 \lambda^g_2} + \frac{\lambda^g_1}{\alpha} \right\} + \frac{\lambda^g_2}{2} \left\{ \frac{(\alpha s)^2 (\hat{\pi} - \hat{\lambda}^{cb})}{(\alpha s)^2 + \phi^2 \lambda^g_2} \right\} > 0 \tag{A40}
\]

irrespective of the value of \( \lambda^{cb} \). But if the government and central bank choose \( \delta = 0 \) and \( \lambda^{cb} = 0 \) respectively, we get

\[
EL^g = \frac{\gamma^2 s^2 [\alpha (\hat{\pi} - \hat{\lambda}^{cb}) + \lambda^g_1]}{2 \phi^2 \lambda^g_2} \tag{A41}
\]

and

\[
EL^{cb} = 0. \tag{A42}
\]

There is no guarantee that (A41) is less than (A39), although it is certain if \( \hat{\lambda}^{cb} \rightarrow \hat{\pi} \) and \( \lambda^g > (\alpha s)^2 / (2\phi^2) \). Hence the monetary authorities can guarantee their preferred solution if the government has a commitment to fiscal sustainability, and if the central bank sets its own inflation target close enough to the government’s not to trigger an override to \( \delta = 1 \). That solution then generates \( \pi^* = \hat{\pi}^{cb} \), and \( E r^* = \gamma s [\alpha (\hat{\pi}^{cb} - \hat{\pi}) - \lambda^g_1] / \phi \lambda^g_2 < 0 \).
APPENDIX C: WHEN WILL GOVERNMENTS BLOCK TARGET INDEPENDENCE?

a) Under fiscal leadership, target independence will always be blocked as is evident from table 1: $EL^g$ is greater using the values in row 5 than using the values in row 3.

b) Under monetary leadership, it will be blocked if the value of (A36) is less than the value of (A41):
\[
\alpha^2 \frac{\gamma^2 s^2}{\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g} < \frac{\gamma^2 s^2 [\alpha(\hat{\pi} - \hat{\pi}^{cb}) + \lambda_1^g]^2}{2\phi^2 \lambda_2^g}.
\]
If $\hat{\pi}^{cb} \rightarrow \hat{\pi}$, this inequality holds when
\[
\frac{\alpha^2}{\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g} < \frac{1}{\phi^2 \lambda_2^g};
\]
and the latter certainly holds if $\alpha < 1$.

If $\hat{\pi}^{cb} < \hat{\pi}$ then the right hand side of the original inequality is increased further.

c) Under simultaneous decision making, target independence will be blocked if
\[
\left(\frac{\lambda_1^g}{\alpha}\right)^2 \left(\frac{\alpha^2 \gamma^2 s^2}{\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g}\right) < \left(\hat{\pi}^{cb} - \hat{\pi}\right)^2 + \lambda_2^g \gamma^2 s^2 \left[\alpha(\hat{\pi}^{cb} - \hat{\pi}) - \lambda_1^g\right]^2 / (\phi \lambda_2^g),
\]
or
\[
0 < \left(1 + \frac{(\alpha \gamma s)^2}{\phi}\right) \left(\hat{\pi}^{cb} - \hat{\pi}\right)^2 - \frac{2\lambda_1^g \alpha \gamma^2 s^2}{\phi} (\hat{\pi}^{cb} - \hat{\pi}) + \frac{(\lambda_1^g \gamma s)^2 (\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g - \phi)}{\phi (\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g)}.
\]
The exact conditions for this inequality are complicated. Simple sufficient conditions are:

i) $\lambda_1^g < 1$ (because $\gamma^2 s^2$ will be small, 0.02 or less in most cases) when $\lambda_2^g$ is not too small;

ii) $\hat{\pi} - \hat{\pi}^{cb} > (\lambda_1^g \lambda^{cb*})^{1/2} / \alpha$, where $\lambda^{cb*}$ is the optimal value of $\lambda^{cb}$ from (10) when $\delta=0$;

(iii) $\phi^2 \lambda_2^g > 1$; or (iv) $\hat{\pi} - \hat{\pi}^{cb} > \lambda_1^g / [2\alpha(\alpha^2 \gamma^2 s^2 + \phi^2 \lambda_2^g)]$.

For convenience the main text takes the first of these conditions, which requires only some commitment to sustainable fiscal policies by the government. Each of these results has been obtained by inserting the outcomes in table 1 into $EL^g$ at (4).